8.5 Solving Exponential Equations

A One-to-one property The exponential function is *one-to-one* function. So: $a^x = a^y \quad \Leftrightarrow \quad x = y$ $a > 0, a \neq 1, x \in R, y \in R,$	Ex 1. Solve the following exponential equations. a) $2^x = 64$ b) $10^{2x-3} = 0.0001$ c) $2^{-x} = \sqrt[5]{16}$ d) $8^x = \sqrt[3]{0.0625}$
B Change of Variable Sometimes, *changing of the variable* may help solving the exponential equation. For example: $a^x = y; \quad y > 0$	
Ex 2. Use the change of variable method to solve each of the following exponential equations. a) $2^x + 2^{-x} = 4.25$ b) $5 \cdot 2^x - 4^x + 24 = 0$	c) $\dfrac{2^x - 2^{-x}}{2^x + 2^{-x}} = -\dfrac{63}{65}$ d) $2^{x+1} + 2^{2x} = 2^x + 2 + \sqrt{2}$

C Logarithms Sometimes, *logarithms* are needed in order to solve exponential equations.	Ex 3. Solve each equation using logarithms. a) $2^{3x-1}=5$ b) $3^{x-1}=4^{x+1}$
D Applications Many *applications* are related to solving exponential equations.	
Ex 4. A species of bacteria doubles each 10 minutes. The initial number of bacteria is 200. a) Find the exponential function describing the bacteria population growth. b) Find the bacteria population after one hour. c) Find the time (in minutes) after which the bacteria population is 123456.	Ex 5. A 100 g sample of plutonium-238 has a half-life of 88 years. a) Find the exponential function describing the radioactive decay. b) Find the mass of radioactive source after 10 years. c) Find the time (in years) after which the mass of the radioactive source will be $3.21\,g$.

Reading: Nelson Textbook, Pages 480-484
Homework: Nelson Textbook, Page 485: #5, 7, 8, 10, 12, 14, 15, 16, 17

